Pensacola Catholic High School
Math Department
Summer Packet | For Students Entering AP Calculus AB

Dear Parent/Guardian and Student,

The Math Department at Pensacola Catholic High will require the completion of a summer packet for each student entering an Honors math course. The problems on this packet are due the second day of Math class (Tuesday, August 10th or Wednesday, August 11th depending on your course schedule). The packet has two purposes: (1) to help you retain the math knowledge you've gained in your previous math classes, and (2) to get a sense of what we expect you to know going into your next class. Here are some tips for working through the packet:

1. We encourage you to work on this packet throughout the summer rather than doing the entire packet at the start or end of the summer. That way you keep the topics you learned fresh in your mind. Do not wait until the last minute to complete this packet!
2. You should complete every problem on the packet and show your work on each problem. Use extra paper if absolutely needed, clearly identifying each problem. All work should be neat, complete, and organized. No problem should be left blank, and no work means no credit.
3. You should not feel obligated to hire an outside tutor. We will spend the first week reviewing material that is necessary. However, you will be tested at the end of the first week on all material in the packet. If you are struggling with the packet, there are free resources, like Khan Academy, that can help.
4. Calculators are allowed for the completion of this packet, but please do not rely on your calculator for the answer. Your assessment will be taken without a calculator.
5. This packet will be graded for correctness and will be one of the first grades of Quarter 1.

Enjoy your Summer and Best of Luck!!
Mrs. Gottstine

AP Calculus AB Summer Packet

The following formulas and identities will help you complete this packet.

Additionally, students are expected to know ALL of these by memory for the course.

Linear forms: Slope-intercept: $y = mx + b$ Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$ Horizontal line: $y = b$ (slope = 0)

Vertical line: $x = a$ (slope is undefined)

Parallel → Equal slopes Perpendicular → Slopes are opposite reciprocals

Quadratic forms: $y = ax^2 + bx + c$ $y = a(x-h)^2 + k$ $y = a(x-p)(x-q)$

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin(2x) = 2\sin x \cos x$ $\cos(2x) = \cos^2 x - \sin^2 x$

$\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$ $= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

Exponential Properties: $x^a \cdot x^b = x^{a+b}$ $(xy)^a = x^a y^a$ $x^0 = 1$ for all $x \neq 0$

$\frac{x^a}{x^b} = x^{a-b}$ $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^b}$ $\sqrt[b]{x^n} = x^{n/b}$ $x^{-n} = \frac{1}{x^n}$

Logarithms: $y = \log_a x$ is equivalent to $a^y = x$

Logarithmic Properties: $\log_b mn = \log_b m + \log_b n$ $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^p) = p \cdot \log_b m$ If $\log_b m = \log_b n$, then $m = n$ $\log_a n = \frac{\log_b n}{\log_b a}$

AP Calculus AB Summer Packet

For #1-10, write an equation for each line in point-slope form.

1. Containing $(4, -1)$ with a slope of $\frac{1}{2}$

2. Crossing the x-axis at $x = -3$ and the y-axis at $y = 6$

3. Containing the points $(-6, -1)$ and $(3,2)$

4. Write an equation of a line passing through $(5, -3)$ with an undefined slope.

5. Write an equation of a line passing through $(-4,2)$ with a slope of 0.

6. Write an equation in point-slope form passing through $(0,5)$ with a slope of $\frac{2}{3}$.

7. Write an equation of a line passing through $(2,8)$ that is parallel to $y = \frac{5}{6}x - 1$.

8. Write an equation of a line passing through $(4,7)$ that is perpendicular to the y-axis.

9. Write an equation of a line with an x-intercept of $(2,0)$ and a y-intercept of $(0,3)$.

10. Write an equation of a line passing through $(6, -7)$ that is perpendicular to $y = -2x - 5$.

For #11-18, solve each equation for x. Note that some equations with have a specific value, but most will have a solution for x in terms of other variables. For example: $x = \frac{a+b}{c}$ *would be a solution.*

11. $x^2 + 3x = 8x - 6$

12. $\frac{2x-5}{x+y} = 3 - y$

13. $3xy + 6x - xz = 12$

14. $A = ax + bx$

15. $cx = vx$

16. $r = t - x(z - y)$

17. $\frac{3+x}{5-x} = 6 + y$

18. $\frac{y+2}{4-x} = 4(2 - z)$

For #19-24, solve each quadratic by factoring.

19. $x^2 - 4x - 12 = 0$

20. $x^2 - 6x + 9 = 0$

21. $x^2 - 9x + 14 = 0$

22. $x^2 - 36 = 0$

23. $9x^2 - 1 = 0$

24. $4x^2 + 4x + 1 = 0$

For #25-29, evaluate the following knowing that $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$.

25. $f\left(\frac{1}{2}\right) =$

26. $g(-2) =$

27. $f(1) + g(0) =$

28. $f(0) \cdot g(0) =$

29. $\frac{g(-6)}{f(-6)} =$

Recall function composition notation $(f \circ g)(x)$ is the same thing as $f(g(x))$.

For #30-39, use $f(x) = x^2 - 1$, $g(x) = 3x$, *and* $h(x) = 5 - x$ *to find each composite function.*

30. $(f \circ g)(x) =$

31. $(g \circ f)(x) =$

32. $(f \circ f)(4) =$

33. $(g \circ h)(-4) =$

34. $(f \circ (g \circ h))(1) =$

35. $(g \circ (g \circ g))(5) =$

36. $f(g(x-1)) =$

37. $g(f(x^3)) =$

38. $\frac{f(x+h)-f(x)}{h} =$

39. The expression in the previous problem is very significant and important in Calculus. Think back to Pre-Calculus… what is the name of that expression?

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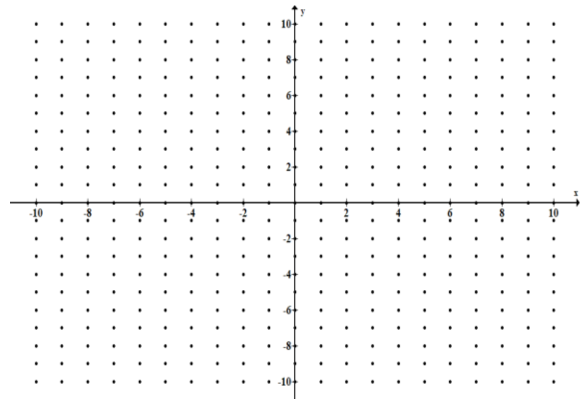
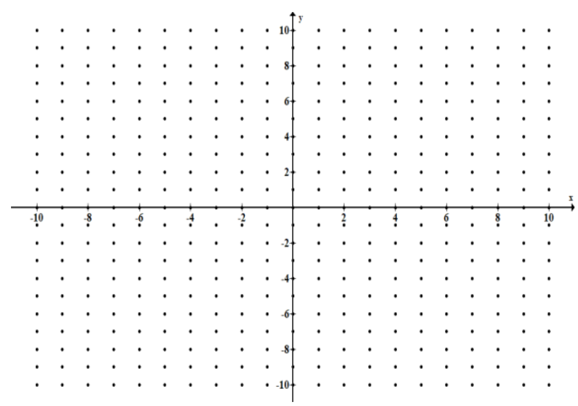
For #40-42, graph each piecewise function.

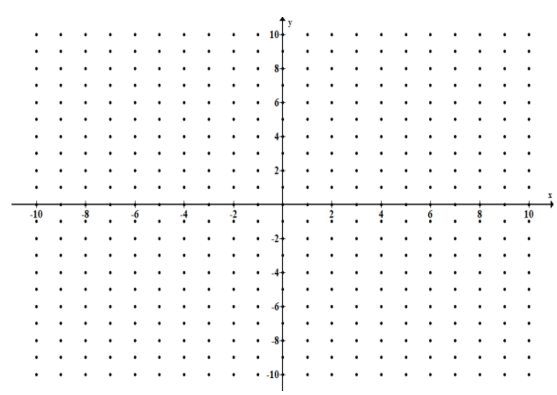
40. $f(x) = \begin{cases} x+3 & ; \ x < 0 \\ -2x+5 & ; \ x \geq 0 \end{cases}$



41. $g(x) = \begin{cases} \frac{1}{2}x & ; \ -4 \leq x \leq 2 \\ 2x-3 & ; \ x > 2 \end{cases}$



42. $h(x) = \begin{cases} |x| & ; \ x \leq 1 \\ 2-|x-2| & ; \ x > 1 \end{cases}$



An ***exponential equation*** *is an equation in which the variable is in the exponent. To solve an exponential equation, you must use a logarithm to solve it.*

For #43-47, solve each exponential equation, rounding answers to the nearest thousandth. Note that some equations can be solved by writing each side as the same base instead of using a logarithm.

43. $5^x = \frac{1}{5}$

44. $6^x = 1296$

45. $6^{2x-7} = 216$

46. $5^{3x-1} = 49$

47. $10^{x+5} = 125$

For #48-51, simplify each expression without the use of a calculator.

48. $e^{\ln 4} =$

49. $e^{2\ln 3} =$

50. $\ln e^9 =$

51. $5\ln e^3 =$

For #52-57, solve each equation using natural logarithm. Round answers to the nearest thousandth.

52. $e^x = 34$

53. $3e^x = 120$

54. $e^x - 8 = 51$

55. $\ln x = 2.5$

56. $\ln(3x - 2) = 2.8$

57. $\ln(e^x) = 5$

For #58-66, find each exact value of the expression using the Unit Circle. ***NO CALCULATOR!***

58. $\sin 120° =$ _________

59. $\cos \frac{11\pi}{6} =$ _________

60. $\tan 225° =$ _________

61. $\sin\left(-\frac{2\pi}{3}\right) =$ ________

62. $\sin 150° =$ _________

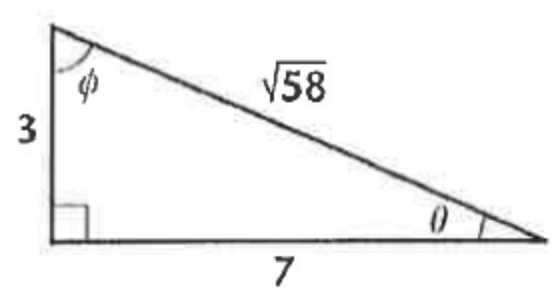
63. $\tan \frac{7\pi}{4} =$ _________

64. $\csc\left(\frac{\pi}{4}\right) =$ ________

65. $\sec(-210°) =$ ______

66. $\cot\left(\frac{5\pi}{4}\right) =$ _________

For #67-74, evaluate each trigonometric expression using the right triangle provided.



67. $\sin\theta =$ _________

68. $\cos\theta =$ _________

69. $\tan\phi =$ _________

70. $\csc\phi =$ _________

71. $\sec\theta =$ _________

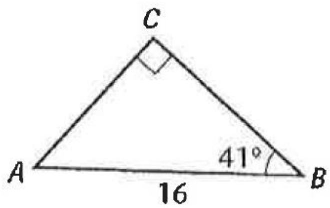
72. $\cot\theta =$ _________

73. $\sin\phi =$ _________

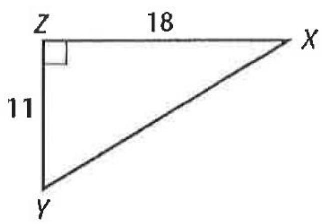
74. $\sec\phi =$ _________

For #75-76, solve each triangle, rounding all angles and sides to the nearest thousandth. Recall that "solve a triangle" means to find all missing sides and angles.

75.



76.



For #77-84, evaluate each inverse trigonometric function. ***NO CALCULATOR!***

77. $\sin^{-1}\left(\frac{1}{2}\right) =$ ________

78. $\sin^{-1}(-1) =$ ________

79. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ ________

80. $\tan^{-1}\left(\sqrt{3}\right) =$ ________

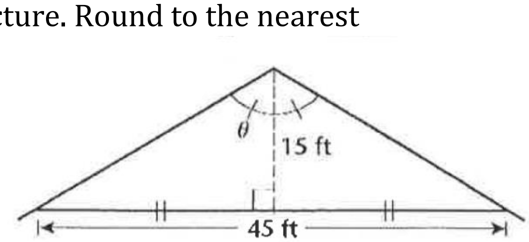
81. $\tan^{-1}(-1) =$ ________

82. $\tan\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) =$ ________

83. $\sin\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right) =$ _______

84. $\sin^{-1}(\cos(0)) =$ ________

85. Find the angle at the peak of the roof, as shown in the picture. Round to the nearest thousandth.



86. Explain how the graph of $f(x)$ and $f^{-1}(x)$ compare.

Recall that to find an inverse of a function, simply switch the x and y and solve for y. We use the notation $f^{-1}(x)$ to define the inverse of $f(x)$.

For #87-89, find the inverse of each function.

87. $g(x) = \frac{5}{x-2}$

88. $f(x) = \frac{x^2}{3}$

89. $y = \sqrt{4-x} + 1$

90. If the graph of $f(x)$ has the point $(2,7)$, then what is one point on the graph of $f^{-1}(x)$?

For #91-94, convert the inequalities in to ***interval notation****. For example, $x > 3$ becomes $(3, \infty)$.*

91. $1 < x \leq 10$

92. $x < 0$ or $x \geq 4$

93. $x \geq -2$

94. $x \geq 4$ and $x > 10$

For #95-100, find the domain and range of each function. Write the answer in interval notation. Confirm your answer by graphing the function in your graphing calculator.

95. $f(x) = \sqrt{x+5}$

D: ____________________ R: ____________________

96. $f(x) = x^2 - 5$

D: ____________________ R: ____________________

97. $f(x) = \frac{1}{x+7}$

D: ____________________ R: ____________________

98. $f(x) = \frac{5}{x^2+1}$

D: ____________________ R: ____________________

99. $f(x) = \sqrt{x^2+5}$

D: ____________________ R: ____________________

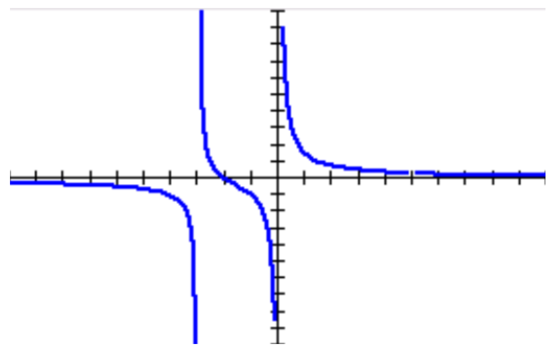
100. $g(x) = x^3 + 2x - 7$

D: ____________________ R: ____________________

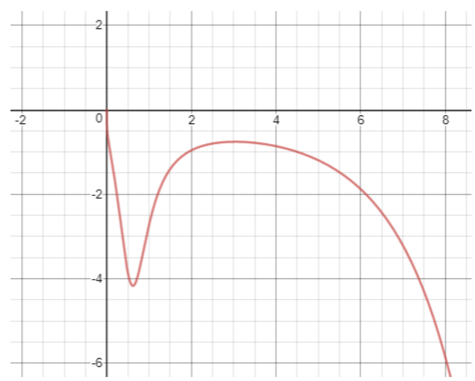
For #101-103, answer the question by referring to the function and its graph.

101. State the domain and range of $f(x) = \frac{2x^2-6x-20}{x^3-2x^2-15x}$

Hint: There is a hole.



102. Consider the function $f(x) = \frac{e^x}{\log x - x^3}$.

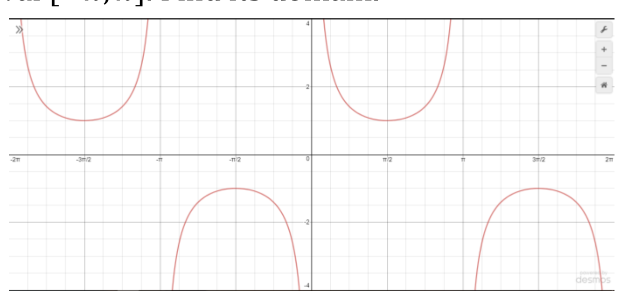


Find the maximum and minimum y-value of the function.

State the domain of $f(x)$.

State when the function is increasing and decreasing (write in interval notation).

103. Consider the function $f(x) = \csc x$ on the interval $[-\pi, \pi]$. Find its domain.



The difference quotient is defined to be $\frac{f(x+h)-f(x)}{h}$ *and is a core concept for the development of calculus. For #104-107, find the difference quotient of each function.*

104. $f(x) = 9x + 3$

105. $f(x) = 5 - 2x$

106. $f(x) = x^2 - 3x$

107. $f(x) = \frac{2}{x+1}$